

Test 3: Projectiles

NAME: KEY
2012

Equations and Constants:

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

$$v = \frac{dx}{dt}$$

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

$$a = \frac{dv}{dt}$$

$$\bar{v} = \frac{1}{2}(v_i + v_f)$$

$$|g| = 10 \text{ m/s}^2$$

$$x = \frac{1}{2}at^2 + v_i t + x_i$$

$$v = at + v_i$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$R = \frac{v^2 \sin 2\theta}{g}$$

$$a_c = \frac{v^2}{r}$$

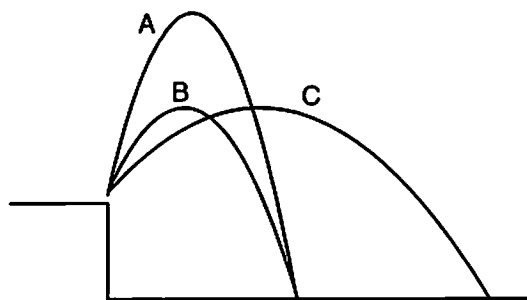
$$2\sin\theta \cos\theta = \sin 2\theta$$

Multiple Choice: Choose the letter of the best answer. 3 points each.

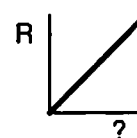
1. B A fruitcake is fired across a level field with some initial velocity. If it were fired again with the same initial angle but twice the initial speed, how much farther horizontally would the fruitcake travel?
- It would travel twice as far as the first fruitcake.
 - It would travel four times further than the first.
 - It would end up in the same place as the first.
 - It would only travel half as far as the first.
 - None of those are correct.

Questions 2 and 3 refer to the following:

Three projectiles are fired from the edge of a cliff. The paths of the projectiles are shown in the diagram to the right. A and B land in the same place. B and C has the same maximum height.



2. E Which projectiles were in the air the same amount of time?
- A.
 - B.
 - C.
 - A & B.
 - B & C.
3. A Which projectile had the smallest horizontal velocity?
- A.
 - B.
 - C.
 - A & B.
 - can't tell.
4. D Back in his prep school days, Mitt did an experiment firing projectiles across a level table. He varied the initial launch angle, but kept the initial speed the same. He measured the resulting ranges, and eventually produced the linear graph shown to the right. Which of the following could have been the horizontal axis?
- θ .
 - $\sin\theta$.
 - $\sin^2\theta$.
 - $\sin 2\theta$.



Questions 5 and 6 refer to the following:

Paul and Joe take turns kicking mudballs in the air over level ground. The mudballs are all kicked with the same initial speed, but different angles.

5. E The mudball in the air the longest will be the one kicked with an initial angle of
- 0° .
 - 30° .
 - 45° .
 - 60° .
 - 90° .
6. C The mudball that travels the farthest horizontally will be the one kicked with an initial angle of
- 0° .
 - 30° .
 - 45° .
 - 60° .
 - 90° .

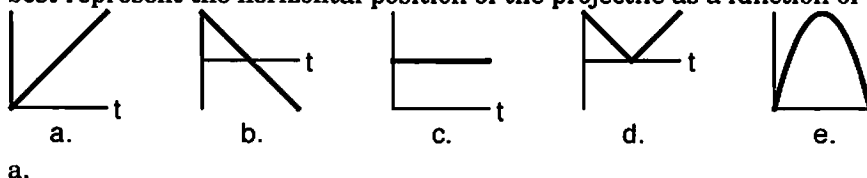
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7. D Projectile A has an initial velocity of $10\mathbf{i} + 20\mathbf{j}$ m/s while projectile B has an initial velocity of $20\mathbf{i} + 10\mathbf{j}$ m/s. They are both launched over level ground. Which of the following statements about the projectiles is true?
- I. They land in the same place. ✓
 II. Projectile A was in the air twice as long as projectile B. ✓
 III. Projectile A went twice as high as projectile B.
- a. I only. b. II only. c. III only. d. I & II only. e. II & III only.

Questions 8 to 10 refer to the following:

On the strange planet Hofstra, a rock is tossed across level ground with an initial velocity of $3\mathbf{i} + 4\mathbf{j}$ m/s. It took 2 seconds for the rock to reach its maximum height.

8. C How far away horizontally did the rock land?
 a. 8 m. b. 10 m. c. 12 m. d. 16 m. e. 20 m.
9. B What was the acceleration due to gravity on Hofstra?
 a. 1 m/s^2 . b. 2 m/s^2 . c. 2.5 m/s^2 . d. 8 m/s^2 . e. can't tell.
10. B At its maximum height, how fast was the rock moving?
 a. 0 m/s. b. 3 m/s. c. 4 m/s. d. 5 m/s. e. can't tell.
11. A Back in his prep school days, Obama investigated projectile motion. After examining the motion of a projectile in the air, he made a variety of graphs. Which of the following would best represent the horizontal position of the projectile as a function of time?



Questions 12 and 13 refer to the following:

Cream tangerine A is launched sideways off the top of a building. At the exact same instant, cream tangerine B is dropped from the same height. Both tangerines hit the level parking lot below.

12. C Which tangerine hits the ground first?
 a. A. b. B. c. They hit at the same time.
 d. Can't tell because one of them will be mistaken for a pigeon and eaten by a falcon.
13. A Which tangerine will be going faster when it hits the ground?
 a. A. b. B. c. They hit with the same speed.

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Problem Solving: Show all work. 10 points each.

14. A TV is pushed horizontally with a speed of 3.5 m/s off the edge of a building over a level parking lot. The building is 30 meters high.

a. How far away from the edge of the building does the projectile land? (6)

$$H = \frac{1}{2} g t^2$$

$$30 = \frac{1}{2} (10) t^2$$

$$t^2 = 6 \quad t = \underline{2.45 \text{ s}}$$

$$x = v_x t$$

$$= (3.5)(2.45)$$

$$= \underline{8.57 \text{ m}}$$

b. What is the velocity of the TV just as it hits the ground? (4)

$$v_y = g t = (10)(2.45) = 24.5$$

$$\vec{V}_p = 3.5 \hat{i} - 24.5 \hat{j} \text{ m/s}$$

15. A projectile is launched across a level field. The initial angle of the projectile was 40° and the projectile was in the air for 3.2 seconds. How far away did the projectile land?



$$t_{\text{up}} = 1.6 \text{ sec.}$$

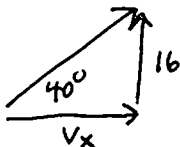
$$0 = -10(1.6) + v_{yi}$$

$$v_{yi} = \underline{16 \text{ m/s}}$$

$$x = v_x t$$

$$= (19.1)(3.2)$$

$$x = \underline{61 \text{ m}}$$



$$\tan 40 = \frac{16}{v_x}$$

$$v_x = \frac{16}{\tan 40} = \underline{19.1 \text{ m/s}}$$

16. Derive an expression for the maximum height of a projectile fired from the ground with an initial velocity of v at an angle of θ . (6 points)

$$v_{yi} = v \sin \theta$$

$$\text{@ top, } v_y = 0 \text{ so}$$

$$0 = -g t + v \sin \theta$$

$$t = \frac{v \sin \theta}{g}$$

$$\text{or } H = \bar{v} t = \left(\frac{v \sin \theta}{2} \right) \left(\frac{v \sin \theta}{g} \right)$$

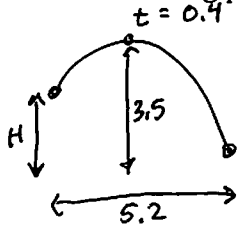
$$H = -\frac{1}{2} g t^2 + v \sin \theta t$$

$$= -\frac{1}{2} g \left[\frac{v \sin \theta}{g} \right]^2 + v \sin \theta \left[\frac{v \sin \theta}{g} \right]$$

$$= \underline{\frac{1}{2} \frac{v^2 \sin^2 \theta}{g}}$$

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17. A ball is launched from some initial, unknown height off the floor. It reaches a maximum height of 3.5 meters from the floor after only 0.4 seconds. It lands 5.2 meters away horizontally from its launching point. What was the initial velocity of the ball?



$t = 2 \times 0.4$

$$0 = -10(0.4) + v_{yi}$$

$$\underline{v_{yi} = 4 \text{ m/s}}$$

to fall 3.5 m:

$$d = \frac{1}{2}at^2$$

$$3.5 = \frac{1}{2}(10)t^2$$

$$.7 = t^2$$

$$\underline{t = .837 \text{ s}}$$

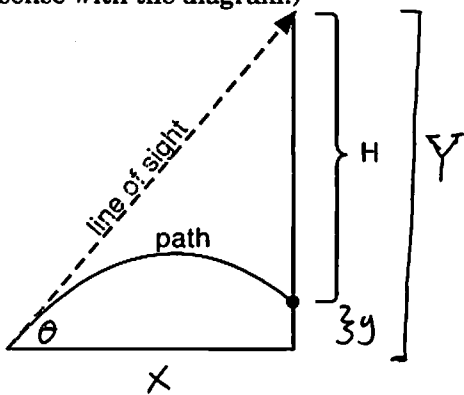
So total time =

$$0.4 + .837 = \underline{1.24 \text{ s}}$$

$$v_x = \frac{x}{t} = \frac{5.2}{1.24} = \underline{4.2 \text{ m/s}}$$

$$\text{so } \underline{\vec{v}_i = 4.2 \hat{i} + 4 \hat{j} \text{ m/s}}$$

18. Imagine firing a projectile with an initial speed of v and initial angle of θ . Show that, after a time of t , the projectile would be exactly $H = \frac{1}{2}gt^2$ below the line of sight. (This makes more sense with the diagram.)



$$Y = x \tan \theta$$

$$y = -\frac{1}{2}gt^2 + v \sin \theta t$$

$$x = v \cos \theta t$$

$$\text{So } H = Y - y$$

$$= x \tan \theta - \left[-\frac{1}{2}gt^2 + v \sin \theta t \right]$$

$$= (v \cos \theta t) \tan \theta + \frac{1}{2}gt^2 - v \sin \theta t$$

$$= vt \sin \theta + \frac{1}{2}gt^2 - v \sin \theta t$$

$$\underline{H = \frac{1}{2}gt^2 \quad \therefore}$$